

S. L. Chuang and J. A. Kong
 Department of Electrical Engineering and
 Computer Science and Research Laboratory
 of Electronics
 Massachusetts Institute of Technology
 Cambridge, Massachusetts 02139

Abstract

Taking into account the penetrable property of metallic gratings, the Smith-Purcell radiation problem is solved and it is shown that when the surface plasmon mode is excited, maximum radiation occurs.

Summary

When an electron beam streams across the surface of a metallic grating, emission of electromagnetic radiation occurs.¹ It was observed experimentally by Smith and Purcell¹ in 1953 and explained with the dipole model formed by the electron and its image.^{1,2} The strength of the dipole oscillates as the electron travels across the grating and thus it radiates. Although this simple model can explain the observed phenomena such as the direction of constructive interference for a specified frequency component, it does not provide a quantitative agreement because the strength of the image can only be obtained in a crude approximation. The exact boundary value problem with the vector nature of the electromagnetic waves is difficult to solve. Two extensive reviews on the emission from charges in periodic structures are given in Refs. 3 and 4, where both qualitative and quantitative approaches to the problems are presented for the work before 1968. For periodic surfaces with sinusoidal profiles, the numerical methods using the boundary integral approach are used to solve the problem exactly^{5,6}. For rectangular gratings, the waveguide mode expansions in the groove region are also used to solve the problem.⁷ However, the previous methods are applied to perfectly conducting gratings only. The extended boundary condition method, which we have used to solve the light diffraction from periodic structures⁸, is applicable in solving the problems of both penetrable and perfectly conducting gratings. The only difference is that in the case of the light diffraction, the incident field is a homogeneous plane wave, and in the case of Smith-Purcell radiation, the incident field is an inhomogeneous (evanescent) plane wave. The energy conservation relation has to be rederived for the interaction of the evanescent wave with the boundary.

We consider the boundary value problem that a line charge is moving with a constant velocity above a layered grating. The field generated by the movement of the line charge in the free space can be obtained in a closed-form expression in the frequency domain. It is evanescent away from the position of the source because the velocity of the charge is less than the velocity of the medium, which is assumed to be free space. This evanescent field is considered as the incident field to the grating, and the diffracted field can be solved using the transition matrix approach⁸. The interaction of the incident field (which does not radiate itself) with the gratings gives the radiation. The radiation field for each frequency component can be calculated. Radiation pattern for particular order of the Floquet wave will be plotted and compared with the results obtained from the boundary integral method⁸.

Since a metal grating at optical frequency becomes penetrable, it is important to take into account the complex permittivity of the metal. It is found that when the surface plasmon mode in the metal-air interface is excited, maximum radiation occurs. The excitation of the surface plasmon mode can be controlled by the period of the grating and the velocity of the electron. As long as one of the Floquet waves is phase-matched to the surface plasmon mode, maximum radiation will happen. As shown in Fig. 1, the radiation pattern shows a sharp peak at the radiation angle where the radiation frequency (which is dependent on the radiation angle) satisfies the phase matching condition.

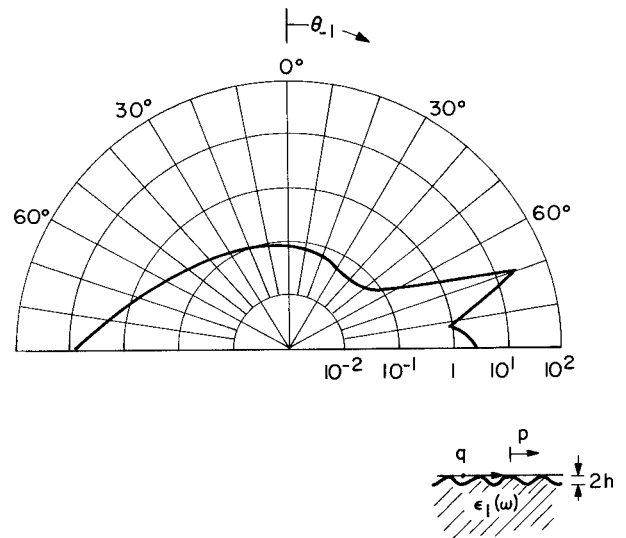


Figure 1

The radiation factor $|R_{-1}|^2$ for the -1 order Floquet wave⁵ as a function of the observation angle for a metallic grating with a complex $\epsilon_1(\omega)$

$$\epsilon_1(\omega) = \epsilon_0 \left[1 + i \frac{\omega_p^2 \tau^2}{\omega \tau (1 - i \omega \tau)} \right],$$

$\tau = 10^{-14}$ sec, $\omega_p \tau = 20\pi$, $P = 2\mu\text{m}$, $h = 0.1\mu\text{m}$ and $v = 0.8c$. The sharp peak at $\theta_{-1} = 71^\circ$ is where the plasmon mode is excited strongly.

References

- 1 S. J. Smith and E. M. Purcell, "Visible light from localized surface charges moving across a grating," Phys. Rev. 92, 1069, 1953.

- 2 K. Ishiguro and T. Tako, "An estimation of Smith-Purcell effect as the light source in the infrared region," *Opt. Acta* 8, 25-31, 1961.
- 3 B. M. Bolotovskii and G. V. Voskresenskii, "Diffraction radiation," *Usp. Fiz. Nauk* 88, 209-251, 1966 [*Sov. Phys. Uspekhi* 9, 73-96, 1966].
- 4 B. M. Bolotovskii and G. V. Voskresenskii, "Emission from charged particles in periodic structures," *Usp. Fiz. Nauk* 94, 377-416, 1968 [*Sov. Phys. Uspekhi*, 143-162, 1968].
- 5 P. M. van den Berg, "Smith-Purcell radiation from a line charge moving parallel to a reflection grating," *J. Opt. Soc. Am.* 63, 689-698, 1973.
- 6 P. M. van den Berg, "Smith-Purcell radiation from a point charge moving parallel to a reflection grating," *J. Opt. Soc. Am.* 63, 1588-1597, 1973.
- 7 P. M. van den Berg and T. H. Tan, "Smith-Purcell radiation from a line charge moving parallel to a reflection grating with rectangular profile," *J. Opt. Soc. Am.* 64, 325-328, 1974.
- 8 S. L. Chuang and J. A. Kong, "Wave scattering from periodic rough surfaces," *Proc. IEEE* 69, 1132-1144, 1981.